Bribery Under Partial Information*

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ABSTRACT

In bribery an external agent tries to alter the outcome of an election by changing some voters' votes. Usually, when investigating bribery problems, full information is assumed, i.e., the manipulative agent knows the set of candidates, each voter's votes and the voting rule used. In this paper, we formally introduce different structures of partial information, we show the connections between them and existing notions, define bribery under partial profiles, and examine the complexity of bribery under partial information for the k-Approval and Veto.

Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multiagent systems*

General Terms

Algorithms, Economics, Theory

Keywords

computational social choice, voting, algorithms and complexity

1. INTRODUCTION

Voting provides a useful method for collective decision making and preference aggregation, and such has applications in politics, economics, and computer science. Usually, in most of the computer science applications, as for example in the design of recommender systems [23], planning [16], machine learning [27], or ranking algorithms [14], we are dealing with huge data volumes thus it is worth studying the computational aspects of problems related to voting. Since the seminal papers of Bartholdi et al. [3, 4, 5], many have investigated the complexity of voting problems in different settings. Examples for voting problems are the *winner problem*, where for a given election we ask whether a distinguished candidate is the winner, or problems related to insincere behavior in elections, such as manipulation, bribery, and control. In order to make a favorite candidate a winner of an election, in *manipulation* a group of voters cast their votes strategically, in *bribery*, an external agent—the briber—changes some voters' votes, and in *control*, an external agent—the chair—changes the structure of the election (see, e.g., the surveys [6, 20]).

Traditionally, the complexity of voting problems is studied under the full information assumption, i.e., for instance in the bribery problem, it is assumed that the briber knows the set of candidates, the set of voters, each voter's full preferences over the candidates, and the voting rule used. However, there are many real-world settings where we simply do not have full information, for example in some multiagent systems applications, where agents represent their preferences via CP-nets [11]. Recently, a number of papers analyzed the complexity of voting problems under some kind of uncertainty.

In this paper, we present a systematic study of the complexity of bribery in settings where the briber has only partial information regarding the voters' preferences. Our main contributions are:

- We introduce three new notions of partial information. In addition, we study six notions that have been introduced or suggested by others in the literature.
- We show the connections between the nine notions of partial information investigated in this paper.
- We analyze the computational complexity of bribery in k-Approval and Veto under all nine notions of partial information.

Related Work.

First of all, regarding full information, bribery was introduced and studied by Faliszewski et al. [19], where the authors investigated the complexity of bribery for Plurality, Veto, and Approval. The bribery results under full information for all k-Approval and k-Veto, for each $k \geq 2$, were published by Lin [26]. In contrast, we are investigating bribery in k-Approval and Veto in settings with partial votes.

Our paper fits in the line of research on the complexity analysis of manipulation and winner determination under some kind of uncertainty. The first work we have to mention here is by Konczak and Lang [25] introducing the possible/necessary winner problems. In their setting, there is uncertainty regarding the votes (specified as partial orders), and they ask whether a candidate is a winner in *at least one* (*possible winner*) or in *all* (*necessary winner*) possible extension of the votes to full orders. A probabilistic

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variant of the possible/necessary winner problem has been introduced by Bachrach et al. [2]. Our problem is related to the possible/necessary winner problems in a sense that we also consider partial votes with the difference that while Konczak and Lang consider partial information in a form of pairwise comparisons, in our work pairwise comparison is only one type of partial information out of nine we investigate. Furthermore, our problem is specifically related to the necessary winner problem in a way that we require from the briber to make his favorite candidate win under *all* possible completions. The possible winner problem was further studied by Xia and Conitzer [28], Betzler and Dorn [10], and Baumeister and Rothe [9].

Chevaleyre et al. [12] investigated the possible winner problem in a setting, where the voters specify their preferences over a set of candidates and after that some new candidates are added to the election. This setting is basically similar to our information model 1TOS, where we assume that the briber knows each voter's preferences over the same subset of candidates. The main difference to our paper is, that they investigate the possible winner problem.

Our work was motivated by Conitzer et al. [13]: in their model a manipulator has partial information about the votes and they ask if the manipulator can cast a vote to improve the outcome of the election. They do not define a special partial information model, but an information set, which is the set of all possible profiles that can be achieved by completing the partial profiles. Basically, their model is a generalization of all the models considered in this paper.

The possible winner problem for top-truncated, bottom-truncated, and doubly-truncated (which is a special case of our information model 1GAP) preferences was investigated by Baumeister et al. [7]. We will use these preference types in our paper, details are coming up in Section 3.

We remark that there is a line of research dealing with uncertainty regarding the voting rule itself [15, 17, 8] but it is very different from our work.

Organization.

This paper is organized as follows. In Section 2, we recall some basics from voting theory. In Section 3, we introduce the models and problems we are considering in our paper and describe the hierarchy of the partial information models. The results on the complexity of bribery under our partial models in k-Approval and Veto are presented in Section 4. Section 5 concludes the paper.

2. PRELIMINARIES

Formally, an *election* is defined by a pair $E = (C, V)$ where C is a finite set of *candidates* with $|C| = m$ and V is a finite set of *voters*. Each voter v_i is represented via its *preference order* $\succ_i$ over the set of candidates which is a strict linear order. We will simultaneously use the terms *ranking* and preference order. In our constructions, we sometimes also insert one or more disjunctive subsets $A, B \subseteq C$ into such preference orders, e.g., $A \succ_v c \succ_v B$ means that voter v prefers each candidate in A to c and c to each candidate in B (note that according to transitivity, v prefers each candidate in A to each candidate in B). Is the voter clear from the context, we omit the index v and write $c \succ B$ instead. $\overrightarrow{B}$ denotes an arbitrary but fixed ordering of the candidates in B. An *n-voter profile* P on C consists of n strict linear orders $P = (v_1, \ldots, v_n)$.

A *voting rule* $\mathcal{E}$ maps election E to a nonempty set $W \subseteq C$. The candidates in W are said to be the *winners* of the election. Throughout this paper, we assume the nonunique-winner model (i.e., every candidate in W is a winner). We will consider the following voting rules in this paper.

- In k-Approval, each voter assigns one point to his k top ranked candidates and gives zero points to all other candidates. The *score* of a candidate c, denoted by $score(c)$, is the sum of points he receives from each voter. The winners are the candidates with the highest score. In some proofs, we will use the notion $score_{V_1}(c)$, denoting the score of c in V_1. 1-Approval is also known as Plurality.

- In k-Veto each voter gives zero points to his k bottom ranked candidates and gives one point to each other candidates. Let $vscore(c)$ denote the number of vetoes c gets. The candidates with the lowest number of vetoes are the winners. We will write Veto for 1-Veto.

Note that while the number of candidates m is not fixed, k is always fixed. In this paper we suppose that the briber does not know the n-voter profile P for a given election E, but has some partial information, a partial profile P'. How this partial profile P' is represented will be discussed in Section 3 in detail. Let $I(P')$ denote the *information set*, which is the set of all complete n-voter profiles which are not contradicted by P'.

3. PARTIAL INFORMATION MODELS

In this section we introduce and motivate different types of partial information models followed by a discussion how these types relate to each other.

3.1 Types of Partial Information

For each of the types of partial information introduced in the following we specify the structure of data given and how the set of potential rankings is specified. We let $m = |C|$.

► Gaps *(GAPS)*

Our first partial information model handles the case, where the briber only knows fractions of each vote, i.e., there are some blocks in each vote that are fully ranked and there are some blocks, where the briber knows which candidates there are in that block, but has no information on how they are ranked. Examples could be nearly single-peaked elections [18], where for every candidate at least an approximate position is known, or cases where the voter is simply indifferent between alternatives.

Formally, for each vote v we have a partition $C_1^v, \ldots, C_{2m+1}^v$ of the set of candidates and a total order for each C_k^v with k even. Note that possibly $C_k^v = \emptyset$ for some k. A ranking of candidates is in the information set if and only if for each (c, c') with $c \in C_k^v$ and $c' \in C_{k'}^v$, $k' > k$, c is preferred to c' and candidates in C_k^v, k even, are ranked according to the total order given for C_k^v.

Note that if $C_k^v = C_{k+1}^v = \emptyset$ we can drop both partite sets without changing the information set. Therefore, we can restrict ourselves to at most $2m + 1$ partite sets.

▸ One Gap *(1GAP)*

A similar model was introduced by Baumeister et al. [7] as doubly-truncated preferences, where in each vote there are subsets of candidates ranked at the top and at the bottom of the votes, and there is a gap between the top and bottom ranked candidates. We adopt this notion and extend it in a way that we allow the top or bottom ranked candidate set to be empty.

Formally, 1GAP refers to the special case of GAPS with $C^v_k = \emptyset$, for each $k \in \{1, 5, 6, \dots, 2m+1\}$, for each voter v.

▸ Top-truncated Orders *(TTO)*

TTO was introduced by Baumeister et al. [7] and refers to the special case of 1GAP where $C^v_1 = C^v_4 = \dots = C^v_{2m+1} = \emptyset$ for each voter v.

▸ Bottom-truncated Orders *(BTO)*

BTO was also introduced by Baumeister et al. [7] and refers to the special case of 1GAP where $C^v_3 = \dots = C^v_{2m+1} = \emptyset$ for each voter v.

▸ Complete or empty votes *(CEV)*

As suggested by Konczak and Lang [25], we introduce CEV as a special case of TTO with either $C^v_2 = \emptyset$ or $C^v_3 = \emptyset$ for each voter v. Note that this is equivalent to the special case of BTO with either $C^v_1 = \emptyset$ or $C^v_2 = \emptyset$ for each voter v. An example for this model is the case, where new voters join the election from whom the briber has absolutely no information.

▸ Fixed Positions *(FP)*

For each vote v we have a subset of candidates C^v with distinct positions in range between 1 and m assigned. A ranking of candidates is in the information set if and only if each candidate in C^v has the assigned position. An example for this model is the case, where there are three candidates c_1, c_2, and c_3. Candidates c_1 and c_3 have clearly opposing properties such that each voter prefers either favors c_1 most and c_3 least or the other way round. Candidate c_2 is fixed to position 2, then.

▸ Pairwise Comparisons *(PC)*

PC is probably the most natural way of displaying partial preferences. It has been introduced by Konczak and Lang [25] and has been used in many papers since. Formally, for each vote v we have a subset Π^v of $C \times C$. A ranking of candidates is in the information set if and only if for each $(c, c') \in \Pi^v$ c is preferred to c'.

Note that we may restrict Π^v to be anti-symmetric and transitive for matters of convenience.

▸ Totally Ordered Subset of Candidates *(TOS)*

For each voter the briber has the information in a form of a totally ordered subset (for each voter a possibly different subset). Such information can emerge for example in sentiment analysis [24], where the briber can extract information from each voter's previous comments or behavior (for example at giving scores for products bought on *ebay*). Formally, for each vote v we have a subset C^v of candidates and a total order for C^v. A ranking of candidates is in the information set if and only if c is preferred to c' for each pair of candidates (c, c') with $c, c' \in C^k$ and c is preferred to c' according to the given order.

▸ Unique Totally Ordered Subset of Candidates *(1TOS)*

1TOS was first suggested by Konczak and Lang [25] and formally defined by Chevaleyre et al. [12]. 1TOS refers to the special case of TOS where $C^v = C'$ for each voter v with $C' \subseteq C$. A natural example here would be the addition of candidates to an election. The briber knows the voters' preferences over the old candidates, but has no information on how the voters would rank the new ones.

3.2 Problem Definitions

Let $X = \{\text{GAPS}, \text{1GAP}, \text{TTO}, \text{BTO}, \text{CEV}, \text{FP}, \text{PC}, \text{TOS}, \text{1TOS}\}$. In the classic bribery problems (with full information about votes and a voting rule given) the question is whether a briber can change a given number of votes such that his favorite candidate is a winner. We carry over this idea to partial information models.

$\mathcal{E}$-X-Bribery	
Given:	An election (C, V), a designated candidate $c \in C$, a non-negative integer ℓ and partial profile P according to model X.
Question:	Is it possible to make c a winner of the election under $\mathcal{E}$ for each complete profile in $I(P)$ by changing up to ℓ votes?

Note that we leave open in the definition whether the changed votes are fully specified or according to X. However, it is rather easy to see that a fully specified vote is according to each partial information model in X and we can restrict ourselves to fully specified bribed votes: If there is a vote with partial information then each preference order in the information set can be chosen.

3.3 Hierarchy

Theorem 3.1 shows the relations between the partial information models discussed in this paper.

THEOREM 3.1. *The following relations hold:*

(1) $1TOS \subsetneq TOS$.
(2) $CEV \subsetneq TOS$.
(3) $CEV \subsetneq TTO$.
(4) $CEV \subsetneq BTO$.
(5) $TTO \subsetneq 1GAP$.
(6) $BTO \subsetneq 1GAP$.
(7) $1GAP \subsetneq GAPS$.
(8) $1GAP \subsetneq FP$.
(9) $TOS \subsetneq PC$.
(10) $GAPS \subsetneq PC$.

This list is complete in the following sense: Relations that are not listed here and that do not follow from transitivity do not hold in general. The relationship of the partial information models is displayed in Figure 1 as a Hasse diagram.

Proof. We will prove the ten relations first, following by points 11-15 showing incomparability for the remaining cases. Results following immediately from definitions or transitivity are not mentioned explicitly.

1. 1TOS $\subsetneq$ TOS: We have 1TOS $\not\supset$ TOS because every voter ranks an individual subset of candidates in general.

2. CEV $\subsetneq$ TOS: Partial information according to CEV can be represented by partial information according to TOS with $C^v = C$ or $C^v = \emptyset$ for each voter v.

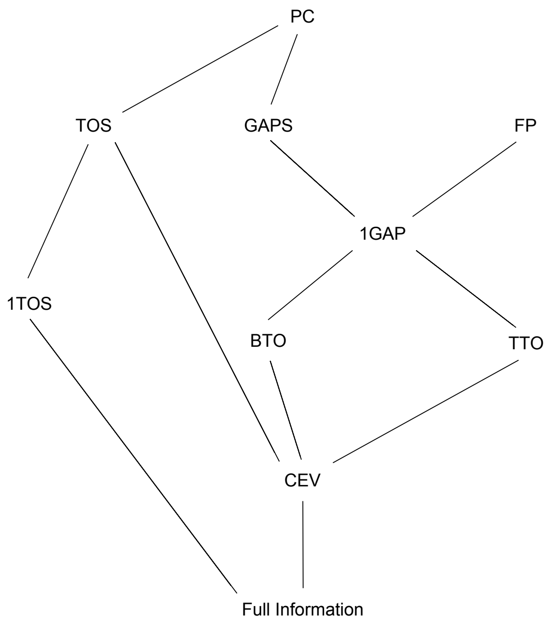

Figure 1: Hasse diagram

The reverse direction is not true in general since $C^v \neq \emptyset$, $C^v \neq C$ cannot be represented in CEV.

3. CEV $\subsetneq$ TTO: We have CEV $\not\supset$ TTO since $C_1^v = \emptyset$, $C_2^v \neq \emptyset$, $C_3^v \neq \emptyset$ cannot be represented in CEV.

4. CEV $\subsetneq$ BTO: We have CEV $\not\supset$ BTO since $C_1^v \neq \emptyset$, $C_2^v \neq \emptyset$ cannot be represented in CEV.

5. TTO $\subsetneq$ 1GAP: We have TTO $\not\supset$ 1GAP since $C_1^v = \emptyset$, $C_2^v \neq \emptyset$, $C_3^v \neq \emptyset$, $C_4^v \neq \emptyset$ cannot be represented in TTO.

6. BTO $\subsetneq$ 1GAP: The same example as for TTO $\not\supset$ 1GAP shows that BTO $\not\supset$ 1GAP.

7. 1GAP $\subsetneq$ GAPS: We have 1GAP $\not\supset$ GAPS since $C_1^v \neq \emptyset$, $C_2^v \neq \emptyset$, $C_3^v \neq \emptyset$ cannot be represented in 1GAP.

8. 1GAP $\subsetneq$ FP: It easy to see that for each candidate in $C_2^v \cup C_4^v$ a unique position is implied by partial information according to 1GAP.

   The reverse direction is not true in general since partial information with only position 2 assigned to a candidate cannot be represented in 1GAP if we have more than two candidates.

9. TOS $\subsetneq$ PC: Let $C^v(l)$ denote the l-most preferred candidate within C^v according to the partial information of type TOS. We can represent the information set by $I(v) = \{(C^v(l), C^v(l)) \mid 1 \leq l \leq |C^v| - 1\}$ as partial information according to type PC.

   The reverse direction is not true in general since partial information $I(v) = \{(c_1, c_2), (c_3, c_4)\}$ cannot be represented in TOS.

10. GAPS $\subsetneq$ PC: Let $C_k^v(l)$ denote the l-most preferred candidate within C_k^v according to the partial information of type TOS for each even k. We can represent the information set by $I(v) = I_1(v) \cup I_2(v)$ where $I_1(v) = \{(c, c') \mid c \in C_k^v, c' \in C_{k+1}^v, k < 2m + 1\}$ and $I_2(v) = \{(C_k^v(l), C_k^v(l)) \mid 1 \leq l \leq |C_k^v| - 1, k \text{ even}\}$ as partial information according to type PC.

    The reverse direction is not true in general since partial information $I(v) = \{(c_1, c_2)\}$ cannot be represented if we have more than two candidates.

11. FP is incomparable to PC, GAPS, TOS, 1TOS: GAPS $\not\subset$ FP since $C_1^v = \{c_1, c_2\}$, $C_2^v = \{c_3\}$, $C_3^v = \{c_4, c_5\}$ cannot be represented in FP. FP $\not\subset$ PC since partial information with only position 2 assigned to a candidate cannot be represented in PC if we have more than two candidates. 1TOS $\not\subset$ FP since partial information $C' \subset C$ and the associated order cannot be represented in FP. The remaining statements follow immediately.

12. GAPS is incomparable to TOS, 1TOS: GAPS $\not\subset$ TOS since $C_k^v = \emptyset$ for each $k > 2$, $C_1^v \neq \emptyset$ and $C_2^v \neq \emptyset$ cannot be represented in TOS. 1TOS $\not\subset$ GAPS since partial information $C' \subset C$ and the associated order cannot be represented in GAPS. The remaining statements follow immediately.

13. TOS is incomparable to 1GAP, BTO, TTO: TOS $\not\subset$ 1GAP since 1TOS $\not\subset$ GAPS. BTO $\not\subset$ TOS since $|C_2^v| = 1$ cannot be represented in TOS. TTO $\not\subset$ TOS since $|C_2^v| = 1$ cannot be represented in TOS. The remaining statements follow immediately.

14. 1TOS is incomparable to 1GAP, BTO, TTO, CEV: CEV $\not\subset$ 1TOS since two votes with full and no information at all cannot be represented simultaneously. 1TOS $\not\subset$ 1GAP since 1TOS $\not\subset$ GAPS. The remaining statements follow immediately.

15. TTO and BTO are incomparable: TTO $\not\subset$ BTO since $2 \leq |C_2^v| \leq |C| - 2$ cannot be represented. BTO $\not\subset$ TTO since $2 \leq |C_2^v| \leq |C| - 2$ cannot be represented.

❑

4. COMPLEXITY RESULTS

Under full information, bribery in most of the prominent voting rules is NP-hard, thus the same holds for bribery under all partial information models. However, under full information, bribery is in P for k-Approval, for all $k \leq 2$, and for k-Veto, for all $k \leq 3$. Table 4.1 shows the results on the complexity of bribery under partial information in k-Approval and k-Veto. Column FI displays the results for the case with full information due to Faliszewski et al. [19] and Lin [26]. Results in italic are hardness results that follow from already existing hardness results for full information. Results in boldface are new.

In this section we will provide the proofs for these results. In some proofs we will use the notion *pscore*(d) (or *pvscore*(d)), which stands for possible score (or possible vetoes) and is the number of approvals (or vetoes) a candidate d gets if we consider the ranking in each voter's information set that is best possible for d with respect to maximization of score (or that is worst possible with respect to minimization of the number of vetoes).

In each NP-hardness proof in this paper we will provide a reduction from the NP-complete problem Exact Cover By 3-Sets (X3C, for short) [22] which is defined as follows.

Exact Cover By 3-Sets	
Given:	A set B with $|B| = 3q$ and a collection $\mathcal{S}$ of 3-element subsets of B.
Question:	Does $\mathcal{S}$ contain an exact cover for B (i.e., a sub-collection $\mathcal{S}' \subseteq \mathcal{S}$ such that every element of B occurs in exactly one member of $\mathcal{S}'$)?

Membership in NP is trivial, we will not mention this in the proofs.

4.1 k-Approval Bribery

As bribery is already hard to solve for many voting rules under full information, bribery under partial information inherits the NP-hardness of bribery under full information (this holds, e.g., for Borda, Maximin, Approval). Thus we only have to regard those voting rules for which bribery can be solved in polynomial time under full information.

4.1.1 Plurality

Theorem 4.1. Plurality-X-Bribery *is NP-complete for* $X \in \{Gaps, 1Gap, FP, TOS, PC, BTO\}$.

Proof. In order to prove NP-hardness, we give a reduction from X3C. To do so, we first give a construction for the reduction, and at the end of the proof we show that the existence of a successful bribery is equivalent to the existence of an exact cover. We prove this only for the structures TOS and BTO. All the remaining structures in X inherit the NP-hardness lower bound from TOS or BTO, as these two are special cases of all the other given structures in X. We will describe the construction for BTO in detail first.

Given an X3C instance $(B, \mathcal{S})$, where $B = \{b_1, \ldots, b_{3m}\}$ and $\mathcal{S} = \{S_1, \ldots, S_n\}$, construct the following bribery instance. The candidate set is $C = B \cup \{c\}$ and c is the distinguished candidate. The bribery limit is m. The set of voters V consists of the following $3mn + 2m - n$ voters of the form:

1. For each i, $1 \leq i \leq n$, there is one voter v_i of the form $S_i \succ \overrightarrow{C \setminus S_i}$. So, we have $C_1^{v_i} = S_i$ and $C_2^{v_i} = C \setminus S_i$.
2. For each j, $1 \leq j \leq 3m$, there are $n + 1 - \ell_j$ voters of the form $b_j \succ \overrightarrow{C \setminus \{b_j\}}$, where $\ell_j = |\{S_i| \ b_j \in S_i\}|$, for all j, $1 \leq j \leq 3m$. Note that these votes are basically complete votes, and since full information is a special case of every structure defined in this paper, these votes can be represented in both structures TOS and BTO.
3. There are $n - m$ voters of the form $c \succ \overrightarrow{C \setminus \{c\}}$.

Note that $score(c) = n - m$ and $pscore(b_j) = n + 1$, for all j, $1 \leq j \leq 3m$. We claim that there exists an exact cover of three-sets if and only if c can be made a winner of the election in all completions.

($\Rightarrow$) Assume there is an exact cover $\mathcal{S}' \subseteq \mathcal{S}$. Changing the corresponding votes in the first voter group such, that the top-ranked candidate in those votes is c makes c a winner under all possible profiles (after the bribery, $score(c) = n$ and $pscore(b_j) = n$, for all j, $1 \leq j \leq 3m$, thus, there exists no candidate who could beat the distinguished candidate c).

($\Leftarrow$) Assume c can be made a winner of the election in all possible completions by bribing at most m voters. Since c cannot reach the score of $n + 1$, each b_j has to lose at least a certain or possible point. This is only possible, if these m votes cover $3m$ candidates, and thus possible points. This, however, is only possible if the bribed voters cover exactly B.

The construction and argumentation is similar for TOS with a slight modification in the representation of the votes in voter set (1). We assume that $S_i = \{b_{i_1}, b_{i_2}, b_{i_3}\}$ denotes the set of possible winner candidates of the given vote v_i and the remaining candidates are excluded from winning this vote. We set $C'_{v_i} = \{b_{i_1}\} \cup (C \setminus S_i)$ as the given totally ordered subset of candidates and let $b_{i_1} \succ \overrightarrow{C \setminus S_i}$. This makes sure that exactly the candidates of S_i are the possible scorers of the vote v_i. ❑

Bribery under the remaining structures is easy in Plurality. In the following, we will provide two polynomial-time algorithms for these structures.

Theorem 4.2. Plurality-TTO-Bribery *is in P.*

Proof. Let V_e denote the set of voters having an empty top set, that is $V_e = \{v \mid C_2^v = \emptyset\}$. The voters in $V \setminus V_e$ have declared their favorite candidate uniquely. It is easy to see that we will not bribe votes definitely favoring c and that bribed votes will favor c. Hence, we have $score(c) = score_{V \setminus V_e}(c) + \ell$ and $pscore(d) = score_{V \setminus V_e}(d) + |V_e| - \ell_d - \ell_e$ after the bribery where ℓ_d is the number of bribed votes in $V \setminus V_e$ that favored d and ℓ_e is the number of bribed votes in V_e.

We can make c a winner if we reach $score(c) \geq pscore(d)$ for each $d \neq c$ while $\ell_e + \sum_{d \in C, d \neq c} \ell_d \leq l$.

Since bribing a vote in V_e is just as costly as bribing any other vote but reduces $pscore(d)$ by one for each candidate $d \neq c$ it is a dominant strategy to bribe $\min\{\ell, |V_e|\}$ votes in V_e.

If $\ell \leq |V_e|$ bribing is done and we can easily check whether it was successful. If $\ell > |V_e|$ it remains to decide which $\ell - |V_e|$ additional votes in $V \setminus V_e$ to bribe. This decision problem can be reduced to Plurality-Bribery under full information which is known to be in P, see [19].

We consider the election $(C, (V \setminus V_e) \cup V'_e)$ where V'_e is a set of $|V_e|$ votes favoring c (representing the bribed votes in V_e). The bribing limit is reduced to $\ell - |V_e|$ since this is the number of votes which can be bribed additional.

To be more precise here, we have to consider the set of votes V'_e implicitly since otherwise the construction of the new election $(C, (V \setminus V_e) \cup V'_e)$ takes pseudo-polynomial time. However, this can be done easily (simply by increasing $score(c)$ by $|V_e|$ explicitly in the algorithm of [19]). ❑

Note that Theorem 4.2 implies that Plurality-CEV-Bribery is in P.

Theorem 4.3. Plurality-1TOS-Bribery *is in P.*

Proof. If $C' = C$ we have the case of bribery under full information which is in P, see [19]. We consider the case $C' \subsetneq C$, therefore, in the following. It is easy to see that bribed votes will favor c. We can make c a winner if we reach $score(c) \geq pscore(d)$ for each $d \neq c$ after bribing no more than ℓ votes. We distinguish two cases.

1. If $C \setminus C' \setminus \{c\} \neq \emptyset$, that is $C \setminus C'$ contains a candidate d, $d \neq c$, we have $score(c) = 0$ before the bribery and

Voting rule	FI	Gaps	FP	TOS	PC	CEV	1TOS	1Gap	TTO	BTO
Plurality	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
2-Approval	P	**NPC**	**NPC**	**NPC**	**NPC**	**P**	**P**	**NPC**	**P**	**NPC**
($\geq$ 3)-Approval	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
($\geq$ 4)-Veto	NPC	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*	*NPC*

Table 1: Summary of the complexity results.

$score(c) = \ell$ after the bribery since each original vote has potential favorite candidate d. Since $pscore(d) = |V| - \ell$ after the bribery we need $\ell \geq |V|/2$. Obviously, $\ell \geq |V|/2$ is also sufficient since no candidate $e \notin \{c, d\}$ can have $pscore(e) \geq |V|/2$ after bribing at least $|V|/2$ and giving their points to c.

2. If $C \setminus C' = \{c\}$ we can easily determine $pscore(d)$ before the bribery for each candidate $d \neq c$. Again, we have $score(c) = \ell$ after the bribery. Note that we reduce $pscore(d)$ by one when bribing a vote where d is the top candidate in C'. Therefore, c can be made a winner if and only

$$\ell \geq \sum_{d \in C'} \max\{0, pscore(d) - \ell\}.$$

Moreover, for each $d \neq c$ we can easily determine the votes to be bribed by choosing $\max\{0, score_{(C',V)}(d) - \ell\}$ arbitrary votes having d as favorite candidate in C'.

❑

4.1.2 2-Approval

Theorem 4.4. 2-Approval-X-Bribery *is NP-complete for* $X \in \{Gaps, 1Gap, FP, TOS, PC, BTO\}$.

Proof. It suffices to show hardness only for $X \in$ {TOS, BTO}. In order to prove NP-hardness, we give a reduction from X3C. To do so, we first give a construction for the reduction, and then we argue that the existence of a successful bribery is equivalent to the existence of an exact cover. We first show the proof for the BTO structure and will describe the necessary small changes for the TOS structure at the end of the proof.

Given an X3C instance $(B, \mathcal{S})$, where $B = \{b_1, \ldots, b_{3m}\}$ and $\mathcal{S} = \{S_1, \ldots, S_n\}$, construct the following bribery instance. The candidate set is

$$C = B \cup \{c\} \cup \{d_1, \ldots, d_{3mn+2m-2n}\}$$

and c is the distinguished candidate. The bribery limit is m. The set of voters V consists of the following $3mn + 2m - n$ voters of the form:

1. For each i, $1 \leq i \leq n$, there is one voter v_i with $C_1^{v_i} = S_i$ and $C_2^{v_i} = C \setminus S_i$.
2. For each j, $1 \leq j \leq 3m$, there are $n+1-\ell_j$ voters who approve of b_j and exactly one of the dummy candidates d_k, $1 \leq k \leq 3mn+3m-3n$, where $\ell_j = |\{S_i | \, b_j \in S_i\}|$, for all j, $1 \leq j \leq 3m$. The other candidates are ranked behind them in arbitrary, but fixed order. As these votes are complete, they can be written in terms of BTO.
3. There are $n - m$ voters of the form approving of c and a dummy candidate d_k, $3mn + 3m - 3n + 1 \leq k \leq 3mn + 2m - 2n$. These votes are complete too.

Note that $score(c) = n - m$ and $pscore(b_j) = n + 1$, for all j, $1 \leq j \leq 3m$. For the dummy candidates, we have $pscore(d_k) = 1$, $1 \leq k \leq 3mn + 2m - 2n$. We claim that there exists an exact cover of three-sets if and only if c can be made a winner of the election in all possible profiles.

($\Rightarrow$) Assume there is an exact cover $\mathcal{S}'$. Changing the corresponding votes in the first voter group such, that the top-ranked candidate in those votes is c makes c a winner under all possible completions. After the bribery, $score(c) = n$ and $pscore(b_j) = n$, for all j, $1 \leq j \leq 3m$, thus, there exists no candidate who beats the distinguished candidate c.

($\Leftarrow$) Assume c can be made a winner of the election in all possible completions by bribing at most m voters. Since c cannot reach the score of $n + 1$, each b_j has to lose at least a certain or possible point. This is only possible, if these m votes cover $3m$ candidates from B, and thus possible points. This, however, is only possible if the bribed voters belong to voter set 1 and cover exactly B.

Under the TOS structure, voter groups 2 and 3 can be left unchanged, since those votes are complete. For the first voter group, we define the voters v_i, $1 \leq i \leq n$, in the following way: We set $S_i = \{b_{i_1}, b_{i_2}, b_{i_3}\}$ and the vote v_i is represented by the totally ordered subset $b_{i_1} \succ b_{i_2} \succ \overrightarrow{C \setminus S_i}$. This way, exactly the candidates from S_i are the possibly but not definitely approved candidates in this vote. The remaining candidates are excluded from being approved by this voter. It is easy to verify, that the scores are identical to the BTO case, and the argumentation is similar too. ❑

In the remaining cases for 2-Approval we will reduce our problems to Capacitated b-Matching which is in P [1, 21]:

Capacitated b-Matching	
Given:	A weighted graph $G = (V, E, \{c_{ij}\}, \{b_i\})$, where b_i is the capacity of vertex i and c_{ij} is the capacity of edge (i, j). All b_i and c_{ij} are integers. Furthermore, an integer K.
Question:	Does G have a capacitated b-matching (i.e., is y_{ij} the number of times edge (i, j) is selected, then $\sum_{j:(i,j) \in E} y_{ij} \leq b_i$ for all i and $y_{ij} \leq c_{ij}$ for each $(i, j) \in E$) with $\sum_{(i,j) \in E} y_{ij} \geq K$?

Theorem 4.5. 2-Approval-TTO-Bribery *is in P.*

Proof. Let V^0 denote the set of voters having an empty top set, that is $V^0 = \{v \mid C_2^v = \emptyset\}$. Furthermore, let $V_c^1 = \{v \mid C_2^v = \{c\}\}$, let $V^1 = \{v \mid |C_2^v| = 1, c \notin C_2^v\}$, let $V_c^2 = \{v \mid |C_2^v| \geq 2, c \in C_2^v\}$, and let $V^2 = \{v \mid |C_2^v| \geq 2, c \notin C_2^v\}$. We can make c a winner if we reach $score(c) \geq pscore(d)$ for each $d \neq c$ after bribing no more than ℓ votes.

It is easy to see that bribed votes will have c among the two most favored candidates. We first detail what we gain from bribing (and changing) a certain vote.

- Bribing a vote in V_c^2 we can reduce the difference of $score(c)$ and $pscore(d)$ by one (for d being the former other top candidate) and we increase the difference by one for one candidate (which we can choose freely among the former losers).
- Bribing a vote in V^2 we can
  - either reduce the difference for one of the former two top candidates by two and for the remaining candidates by one or
  - reduce the difference for both former two top candidates by two, do not change the difference for one of the former losers (which can choose freely), and increase it for the remaining candidates by one.
- Bribing a vote in V_c^1 we do not change the difference for one candidate (which we can choose freely) and reduce it for the remaining candidates by one.
- Bribing a vote in V^1 we reduce the difference for one candidate (which we can choose freely) by one and reduce it for the remaining candidates by two.
- Bribing a vote in V^0 we reduce the difference for one candidate (which we can choose freely) by one and reduce it for the remaining candidates by two.

Now we can see that bribing any vote in $V^0 \cup V^1 \cup V^2$ where we can ensure that the difference is reduced by at least one for each candidate is dominant to bribing any vote in $V_c^1 \cup V_c^2$ where we yield a reduction of at most one per candidate. Hence, we may assume that votes in $V_c^1 \cup V_c^2$ will not be bribed at all since c is a winner if all other votes are bribed. Furthermore, we see that bribing any vote in $V^0 \cup V^1$ where we can ensure that the difference is reduced by two for each candidate but one (which we can choose freely) is dominant to bribing any vote in V^2. Now, we distinguish three cases.

1. If $\ell \geq |V^0 \cup V^1 \cup V^2|$ we simply bribe all votes in $V^0 \cup V^1 \cup V^2$ and c is a winner.
2. If $\ell \leq |V^0 \cup V^1|$ we bribe ℓ arbitrary votes in $V^0 \cup V^1$. We determine $pscore(d)$ for each candidate $d \neq c$ before the bribery, reduce it by ℓ in order to account for the bribed votes, and construct the bribed votes one by one as follows. We give c the top position and the second position goes to one of the candidates having currently the lowest $pscore(d)$. It is easy to see that this procedure keeps the maximum $pscore(d)$ after the bribery as low as possible. Checking whether $score(c) \geq pscore(d)$ for each candidate $d \neq c$ gives us a certificate for a successful bribery or for the fact that c cannot be made a winner
3. If $|V^0 \cup V^1 \cup V^2| > \ell > |V^0 \cup V^1|$ we bribe all votes in $V^0 \cup V^1$ and $\ell - |V^0 \cup V^1|$ votes in V^2. This gives us $score(c) = |V_c^1 \cup V_c^2| + \ell$ after the bribery.

   After the bribery, we have for $d \neq c$, $pscore(d) = pscore_{V_c^1 \cup V_c^2}(d) + score_{\bar{V}^2}(d) + b_d$, where $\bar{V}^2$ is the set of votes in V^2 not bribed and b_d is the number of points given to d by bribed votes.

   Note that there are ℓ points from bribed votes that we can distribute to candidates except c arbitrarily.

   Hence, what we ask for is a set $\bar{V}^2$ of $|V^0 \cup V^1 \cup V^2| - \ell$ votes in V^2 not to be bribed such that each candidate $d \neq c$ has $pscore_{V_c^1 \cup V_c^2}(d) + score_{\bar{V}^2}(d) \leq |V_c^1 \cup V_c^2| + \ell$. Note that $pscore_{V_c^1 \cup V_c^2}(d) + score_{\bar{V}^2}(d)$ can be described as the $pscore$ value for each $d \neq c$ without accounting for the ℓ points from bribed votes.

   We find $\bar{V}^2$ employing graph $G = (V, E)$ as follows. We have $V = C \setminus \{c\}$. Furthermore, we have the set of edges
   $$E = \{\{d, d'\} \mid d, d' \in C \setminus \{c\}, d \neq d'\}.$$
   Each edge $e = \{d, d'\}$ has capacity u_e of the multiplicity with which d and d' appear as first two candidates (in arbitrary order) in a vote in V^2. Each node d has a capacity of $\left(|V_c^1 \cup V_c^2| + \ell\right) - pscore_{V_c^1 \cup V_c^2}(d)$. We ask for a capacitated b-matching of maximum cardinality. If the cardinality of this matching is at least $|V^0 \cup V^1 \cup V^2| - \ell$ we keep $|V^0 \cup V^1 \cup V^2| - \ell$ arbitrary votes corresponding to edges in the matching and bribe the remaining ones. Otherwise, we cannot make c a winner.

   If we succeeded in finding $\bar{V}^2$, then it remains to distribute the ℓ points from bribed votes to the candidates such that each candidate $d \neq c$ has $pscore_{V_c^1 \cup V_c^2}(d) + score_{\bar{V}^2}(d) + b_d \leq |V_c^1 \cup V_c^2| + \ell$. The latter can be done – if possible at all – by the same mechanism as in 2. Note that given that votes in $V_c^1 \cup V_c^2$ are not bribed and we bribe exactly ℓ votes, the total $pscore$ value after bribery amounts to
   $$\sum_{d \neq c} \left(pscore_{V_c^1 \cup V_c^2}(d) + score_{\bar{V}^2}(d) + b_d\right)$$
   and is a constant. If we succeeded also in distributing the ℓ points from bribed votes we make c a winner. If we did not, we cannot make c a winner.

❑

Note that Theorem 4.5 implies that 2-Approval-CEV-Bribery is in P.

Theorem 4.6. 2-Approval-1TOS-Bribery *is in P.*

Proof. As before, we can make c a winner if we reach $score(c) \geq pscore(d)$ for each $d \neq c$ after bribing no more than ℓ votes. Obviously, a bribed vote will have c among the two top candidates. We assume that $C' \neq C$ since otherwise we have full information and the problem can be solved in polynomial time, see [19]. We distinguish three cases.

1. If $c \notin C'$, then $score(c) = 0$ before the bribery and $score(c) = \ell$ afterwards. Each candidate $d \in C \setminus C'$, $d \neq c$, has $pscore(d) = |V|$ before the bribery and at least $pscore(d) = |V| - \ell$ afterwards. Hence, if $|C \setminus C'| \geq 2$ and $\ell < |V|/2$, c cannot be made a winner.

   If $|C \setminus C'| = 1$ or $\ell \geq |V|/2$ we find a set of votes not to be bribed as follows. We construct a graph $G = (V, E)$.

We have $V = C \setminus \{c\}$. Furthermore, we have the set of edges

$$E = \{\{d, d'\} \mid d, d' \in C', d \neq d'\} .$$

Each edge $e = \{d, d'\}$ has capacity u_e of the multiplicity with which d and d' appear as the first two candidates (in arbitrary order) in C' in a vote. Each node d has a capacity of ℓ. We ask for a capacitated b-matching of maximum cardinality. If the cardinality of this matching is at least $|V| - \ell$ we keep $|V| - \ell$ arbitrary votes corresponding to edges in the matching and bribe the remaining ones. Otherwise, we cannot make c a winner. It remains to detail how to set bribed votes. We distribute the points to be given to candidates other than c by the same mechanism as used in the proof of Theorem 4.5.

2. If $c \in C'$ and $|C \setminus C'| \geq 2$, then $score(c) = 0$ before the bribery and $score(c) = \ell$ afterwards. Again, if $\ell < |V|/2$, c cannot be made a winner. If $\ell \geq |V|/2$ we find a set of votes not to be bribed by using a similar graph as in 1. However, we also have c as a node with infinite capacity. Edges are constructed just as in 1. Again, we ask for a capacitated b-matching of maximum cardinality. If the cardinality of this matching is at least $|V| - \ell$ we are done. We keep $|V| - \ell$ arbitrary votes corresponding to edges in the matching and bribe the remaining ones. Otherwise, we cannot make c a winner. Bribed votes are set as in 1.

3. If $c \in C'$ and $|C \setminus C'| = 1$, let V_c be the set of votes where c has the first position within C'. We can see that by bribing a vote in V_c, we reduce the difference between $score(c)$ and $pscore(d)$ for each candidate $d \neq c$ by at most one while we reduce it by at least one bribing a vote in $V \setminus V_c$. If $\ell \geq |V \setminus V_c|$, then we can make c a winner easily by bribing all votes in $V \setminus V_c$. If $\ell < |V \setminus V_c|$ we bribe only votes in $V \setminus V_c$. Therefore, $score(c) = |V_c|$ before the bribery and $score(c) = |V_c| + \ell$ afterwards. After the bribery we have $pscore(d) \geq |V| - \ell$ for $d \in C \setminus C'$. Hence, if $\ell < \lceil |V \setminus V_c|/2 \rceil$, c cannot be made a winner.

   If $\ell \geq \lceil |V \setminus V_c|/2 \rceil$ we find a set of votes not to be bribed by using a similar graph as in 2. However, node d, $d \neq c$, has capacity of $|V_c| + \ell$. Again, we ask for a capacitated b-matching of maximum cardinality. If the cardinality of this matching is at least $|V| - \ell$ we are done. We keep $|V| - \ell$ arbitrary votes corresponding to edges in the matching and bribe the remaining ones. Otherwise, we cannot make c a winner. Bribed votes are set as in 1. and 2.

❑

4.2 Veto Bribery

THEOREM 4.7. Veto-FP-BRIBERY *is in P.*

Proof. In case there is a vote with $m - 1$ fixed positions we fix the last position, as well. Henceforth, we assume that there are no votes with exactly $m - 1$ fixed positions in the following.

Let V_c be the set of votes where either c is fixed to the last position or c is not fixed to any position and no candidate is fixed to the last position. We then have $pvscore(c) = |V_c|$ before the bribery. Furthermore, before the bribery $vscore(d)$ equals the number of votes where d is fixed to the last position. We can make c a winner if $pvscore(c) \leq vscore(d)$ for each $d \neq c$ after the bribery.

Obviously, a bribed vote will not have c on the last position. We can see that bribing a vote in V_c we reduce the difference between $vscore(d)$ and $pvscore(c)$ for each candidate $d \neq c$ by at least one while we reduce it by at most one bribing a vote in $V \setminus V_c$. Hence, if $\ell \geq V_c$ we simply bribe all votes in V_c. We assume, therefore, that $\ell < V_c$ in the following. We bribe only votes in V_c and we have $pvscore(c) = |V_c| - \ell$ after the bribery. Bribing a vote we are free whom to give the veto to. Hence, we can make c a winner if and only if

$$\ell \geq \sum_{d \in C \setminus \{c\}} \max\{0, pvscore(c) - \ell - vscore(d)\}.$$

❑

THEOREM 4.8. Veto-PC-BRIBERY *is in P.*

Proof. The proof is analogue to the one for Theorem 4.7. The only difference is how $vscore(d)$ and $pvscore(c)$ are determined. Let L^v be the set of candidates who may get a veto by voter v, that is $L^v = \{d \mid \nexists e \in C, (d, e) \in \Pi\}$. Now we have $pvscore(c) = |\{v \mid c \in L^v\}|$ and $vscore(d) = |\{v \mid L^v = \{d\}\}|$. ❑

Note that Theorems 4.7 and 4.8 imply that Veto-X-BRIBERY is in P for $X \in \{\text{Gaps}, \text{1Gap}, \text{FP}, \text{TOS}, \text{PC}, \text{CEV}, \text{1TOS}, \text{TTO}, \text{BTO}\}$.

5. CONCLUSIONS

We have introduced three new partial information models (Gaps, Fixed Positions, and Totally Ordered Subsets of Candidates) and studied six known models. We have shown the relations of all nine partial information models discussed in this paper. Furthermore, we have defined bribery under partial information and investigated the complexity of this problem under k-Approval and Veto. We refer the reader to Table 4.1 for an overview. The first open questions here are the complexity of bribery under all partial information models under the voting rules 2-Veto and 3-Veto. Second, we could define bribery in the sense of the possible winner problem, i.e., the briber's goal would be to make his favorite candidate a winner in at least *one* completion.

Another interesting direction is investigating the complexity of manipulation, control and dominating manipulation (as introduced in [13]) under these partial models. Furthermore, motivated by dominating manipulation, one could define dominating bribery under partial information as the problem, where an election, a partial profile, the briber's preference order, and a nonnegative integer ℓ are given and the question is whether the briber can achieve a better outcome by bribing at most ℓ voters.

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REFERENCES

[1] K. J. Ahn and S. Guha. Near linear time approximation schemes for uncapacitated and capacitated b-matching problems in nonbipartite graphs. In *Proceedings of the Twenty-Fifth Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 239–258. SIAM, January 2014.

[2] Y. Bachrach, N. Betzler, and P. Faliszewski. Probabilistic possible winner determination. In *Proceedings of the 24th AAAI Conference on Artificial Intelligence*, pages 697–702. AAAI Press, July 2010.

[3] J. Bartholdi, C. Tovey, and M. Trick. The computational difficulty of manipulating an election. *Social Choice and Welfare*, 6(3):227–241, 1989.

[4] J. Bartholdi, C. Tovey, and M. Trick. Voting schemes for which it can be difficult to tell who won the election. *Social Choice and Welfare*, 6(2):157–165, 1989.

[5] J. Bartholdi, C. Tovey, and M. Trick. How hard is it to control an election? *Mathematical and Computer Modeling*, 16(8/9):27–40, 1992.

[6] D. Baumeister, G. Erdélyi, E. Hemaspaandra, L. Hemaspaandra, and J. Rothe. Computational aspects of approval voting. In J. Laslier and R. Sanver, editors, *Handbook on Approval Voting*, chapter 10, pages 199–251. Springer, 2010.

[7] D. Baumeister, P. Faliszewski, J. Lang, and J. Rothe. Campaigns for lazy voters: Truncated ballots. In *Proceedings of the 11th International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 577–584. IFAAMAS, June 2012.

[8] D. Baumeister, M. Roos, and J. Rothe. Computational complexity of two variants of the possible winner problem. In *Proceedings of the 10th International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 853–860. IFAAMAS, May 2011.

[9] D. Baumeister and J. Rothe. Taking the final step to a full dichotomy of the possible winner problem in pure scoring rules. *Information Processing Letters*, 112(5):186–190, 2012.

[10] N. Betzler and B. Dorn. Towards a dichotomy for the possible winner problem in elections based on scoring rules. *Journal of Computer and System Sciences*, pages 812–836, 2010.

[11] C. Boutilier, R. Brafman, C. Domshlak, H. Hoos, and D. Poole. CP-nets: A tool for representing and reasoning with conditional ceteris paribus statements. *Journal of Artificial Intelligence Research*, 21:135–191, 2004.

[12] Y. Chevaleyre, J. Lang, N. Maudet, and J. Monnot. Possible winners when new candidates are added: The case of scoring rules. In *Proceedings of the 24th AAAI Conference on Artificial Intelligence*, pages 762–767. AAAI Press, July 2010.

[13] V. Conitzer, T. Walsh, and L. Xia. Dominating manipulations in voting with partial information. In *Proceedings of the 25th AAAI Conference on Artificial Intelligence*, pages 638–643. AAAI Press, August 2011.

[14] C. Dwork, R. Kumar, M. Naor, and D. Sivakumar. Rank aggregation methods for the web. In *Proceedings of the 10th International World Wide Web Conference*, pages 613–622. ACM Press, May 2001.

[15] E. Elkind and G. Erdélyi. Manipulation under voting rule uncertainty. In *Proceedings of the 11th International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 624–634. IFAAMAS, June 2012.

[16] E. Ephrati and J. Rosenschein. A heuristic technique for multi-agent planning. *Annals of Mathematics and Artificial Intelligence*, 20(1–4):13–67, 1997.

[17] G. Erdélyi, E. Hemaspaandra, and L. Hemaspaandra. Bribery and voter control under voting-rule uncertainty. In *Proceedings of the 13th International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 61–68. IFAAMAS, May 2014.

[18] G. Erdélyi, M. Lackner, and A. Pfandler. Computational aspects of nearly single-peaked electorates. In *Proceedings of the 27th AAAI Conference on Artificial Intelligence*, pages 283–289. AAAI Press, July 2013.

[19] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. How hard is bribery in elections? *Journal of Artificial Intelligence Research*, 35:485–532, 2009.

[20] P. Faliszewski and A. D. Procaccia. AI's war on manipulation: Are we winning? *AI Magazine*, 31(4):53–64, 2010.

[21] H. N. Gabow. An efficient reduction technique for degree-constrained subgraph and bidirected network flow problems. In *Proceedings of the fifteenth annual ACM symposium on Theory of computing*, pages 448–456. ACM, 1983.

[22] M. Garey and D. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman and Company, New York, 1979.

[23] S. Ghosh, M. Mundhe, K. Hernandez, and S. Sen. Voting for movies: The anatomy of recommender systems. In *Proceedings of the 3rd Annual Conference on Autonomous Agents*, pages 434–435. ACM Press, May 1999.

[24] U. Grandi, A. Loreggia, F. Rossi, and V. A. Saraswat. From sentiment analysis to preference aggregation. In *International Symposium on Artificial Intelligence and Mathematics*, January 2014.

[25] K. Konczak and J. Lang. Voting procedures with incomplete preferences. In *Proceedings of IJCAI-05 Multidisciplinary Workshop on Advances in Preference Handling*, pages 124–129, 2005.

[26] A. Lin. The complexity of manipulating k-approval elections. In *Proceedings of the 3rd International Conference on Agents and Artificial Intelligence*, pages 212–218, 2011.

[27] L. Xia. Designing social choice mechanisms using machine learning. In *Proceedings of the 12th International Conference on Autonomous Agents and Multiagent Systems*, pages 471–474. IFAAMAS, May 2013.

[28] L. Xia and V. Conitzer. Determining possible and necessary winners given partial orders. *Journal of Artificial Intelligence Research*, 41:25–67, 2011.